Oleksii Nosko

Strength of Structures: Lecture notes and study problems

Preface

This book is a collection of lecture notes and study problems on the subject of *Strength of Materials*, also known as *Mechanics of Materials* or *Mechanics of Deformable Bodies*. It is designed to teach students the most important terms, assumptions, principles and methods of the subject.

The content of the book is organised in the form of ten lectures. Lectures **1**–**6** are dedicated to the problems of tension, compression, torsion and bending. Lectures **7**–**8** deal with the problems of plane stress under combined loading. Lectures **9**–**10** introduce the energy methods for solving statically determinate and indeterminate problems.

Before taking the course of *Strength of Materials*, students should have completed a course of *Mechanics*. In particular, they are expected to know and be able to apply the conditions of static equilibrium and the principles of conservation of energy. Since the solution of study problems requires the application of the methods of differential and integral calculus, students should also have completed a course of *Mathematical Analysis*.

The focus of the book is to help students develop the skills to efficiently schematise, solve and analyse the typical problems of the subject. The study problems considered are categorised into twenty groups **P1**–**P20**, each of which is assigned to a specific lecture. The study problems are formulated both in ‘classical’ form, when the aim is to determine an internal force, displacement, stress or strain in a structure under the action of certain loads, and in ‘engineering’ form, when the aim is to determine the maximum loads or minimum geometric sizes for which the structure retains its load-carrying capacity. The book contains all algorithms and formulae required to solve the study problems. Nonetheless, it should be considered as a basic introduction to the subject. Systematic reading of the comprehensive textbooks, e.g. [1−5], is necessary for a deeper understanding of the subject.

Finally, I would like to express my gratitude to Prof. Heorhiy Sulym at Ivan Franko National University of Lviv, Prof. Pavlo Nosko at Kyiv Aviation Institute and Dr Yurii Tsybrii at Gdansk University of Technology for their kind assistance in the preparation of this book. My sincere thanks also go to my family for their support and patience.

Oleksii Nosko

Contents

Notation

a	coefficient, size, diameter, m
b	coefficient, width, m
d	diameter, diagonal, distance, m
$d_{\max}$	maximum distance from the neutral axis, m
h	height, m
i, j	indices
k	spring constant, N/m; index
m	intensity of a distributed twisting moment, N
n	safety factor; degree of static indeterminacy
$\bar{n}$	unit normal vector; principal direction
q	intensity of a distributed axial force, N/m
r	radius, m
s	section modulus, m^3
t	thickness, m
$\bar{t}$	stress vector, Pa
v	beam deflection, m
w	intensity of a distributed transverse force, N/m
x, y, z	coordinates, m
$x_{\max}$	maximum x-distance from the neutral axis, m
$y_{\max}$	maximum y-distance from the neutral axis, m
A	cross-sectional area, m^2
C	centroid
C_{R}	resultant force couple, Nm
E	elastic modulus, Pa
F	force, N
G	shear modulus, Pa
I	moment of inertia, m^4
I_{xy}	product of inertia, m^4
I_1, I_2	principal moments of inertia, m^4
J	polar moment of inertia, m^4
L	length, m

M	moment, bending moment, Nm
M_i	internal bending moment, Nm
M_max	maximum bending moment, Nm
N	normal force, N
P	axial force, normal force, N
P_i	internal axial force, N
Q	static moment, m^3; dummy load, N or Nm; unit load
R	resultant force, N
T	twisting moment (torque), Nm
T_i	internal twisting moment, Nm
U	strain energy, J
V	shear force, N
V_max	maximum shear force, N
W	work, J
X	redundant reaction, N or Nm
X, Y, Z	centroidal axes, principal axes, principal directions; coordinates, m
α	linear thermal expansion coefficient, 1/°C
γ	shear strain, rad
δ	elongation, displacement, gap, m
ϵ	normal strain
ϵ_T	thermal strain
θ	angle of twist, rad
ν	Poisson’s ratio
π	Pi number, $\pi \approx 3.14$
ρ	radial coordinate, curvature radius, m
σ	normal stress, Pa
σ'	complementary normal stress, Pa
$\sigma_1, \sigma_2, \sigma_3$	principal stresses, Pa
σ_e	equivalent stress, Pa
σ_max	maximum stress, maximum normal stress, Pa
σ_ult	ultimate stress, Pa
σ_w	working stress, Pa

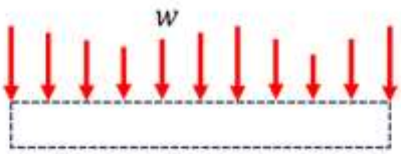
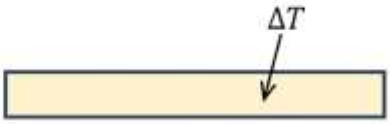
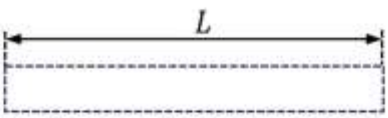
σ_{yp}	yield stress (yield point), Pa
τ	shear stress, Pa
τ'	complementary shear stress, Pa
τ_{max}	maximum shear stress, Pa
τ_{w}	working shear stress, Pa
φ	angle, slope angle, angle of rotation, rad
ΔA	area element, m^2
ΔL	length element, m
ΔT	temperature increase, °C
$\mathrm{atan}(\blacksquare)$	inverse tangent function
$\cos(\blacksquare)$	cosine function
$\max(\blacksquare)$	function returning the maximum value
$\min(\blacksquare)$	function returning the minimum value
$\sin(\blacksquare)$	sine function
$\tan(\blacksquare)$	tangent function
$\overline{\blacksquare}$	vector
$\blacksquare_i$	i-th component of a vector; i-th element of a system
$\blacksquare_{\mathrm{max}}$	maximum value
$\blacksquare_x$	related to the axis x
$\blacksquare_y$	related to the axis y
$\blacksquare_z$	related to the axis z
$\blacksquare_{\mathrm{c}}$	related to the centroid C
$\langle\blacksquare\rangle$	bracket function
NA	neutral axis
J	joule
μm	micrometre = 10^{-6} m
mm	millimetre = 10^{-3} m
m	metre
N	newton
kN	kilonewton = 10^3 N
Pa	pascal = $\mathrm{N/m}^2$
MPa	megapascal = 10^6 pascal

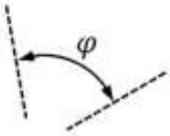
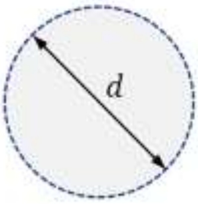
GPa	gigapascal = 10^9 pascal
°C	degree Celsius

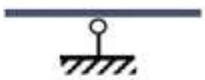
Index

Schematic designations

Designation	Description
	Force vector. $[F] = \mathrm{N}$.
	Force vector perpendicular to the plane and directed towards us. $[F] = \mathrm{N}$.
	Force vector perpendicular to the plane and directed away from us. $[F] = \mathrm{N}$.
	Force couple vector with moment M. $[M] = \mathrm{Nm}$.
	Moment M acting in the plane. $[M] = \mathrm{Nm}$.

	Distributed axial force of intensity q. $[q] = \mathrm{N/m}$.
	Distributed twisting moment of intensity m. $[m] = \mathrm{N}$.
	Distributed transverse force of intensity w. $[w] = \mathrm{N/m}$.
	Thermal load in the form of temperature increase ΔT. $[\Delta T] = {}^\circ\mathrm{C}$.
	Linear size L. $[L] = \mathrm{m}$.

	Angle φ. $[\varphi] = \text{rad}$.
	Radius r. $[r] = \text{m}$.
	Diameter d. $[d] = \text{m}$.
	Circular cross section of diameter d. $[d] = \text{m}$.
	Square cross section of size a. $[a] = \text{m}$.

	Spring of stiffness k. $[k] = \mathrm{N/m}$.
	Gap δ. $[\delta] = \mathrm{m}$.
	Pin-joint connection of two elements that allows their relative rotation in the plane.
	Roller support that prevents the element from translation in the direction perpendicular to the surface.
	Pin-joint support, also called ‘hinge’, which blocks translation of the element in any direction in the plane.

	Fixed support, also called ‘built-in support’, which blocks both translation and rotation of the element.
	Guided support that blocks rotation of the element in the plane.
	Guided support that blocks rotation of the element in the plane and its translation in the direction perpendicular to the surfaces.